While each of these terms after the first, approach 0 as $a=b$, making it appear that from this view one root is $\frac{1}{2}a$ instead of a, as found above; yet by writing the series as follows:

$$1+(a-b)\left[\frac{a}{\sqrt{(2a^2-b^2)}}+\frac{a^2}{(2a^2-b^2)}+\frac{a^3}{(2a^2-b^2)^{\frac{3}{2}}}+\text{ etc.}\right],$$

it is seen that this factor takes the form, in the limit, $1+0\times\infty$; and, therefore, this method is no more capable of yielding a determinate result than is the original expression. EDITOR F.

ALGEBRA.

108. Proposed by GEORGE LILLEY, Ph.D., LL.D., Professor of Mathematics, State University, Eugene, Or.

A gave two notes; one for a dollars at m per cent., and the other for b dollars at n per cent. annual interest. He is to make a monthly payment of c dollars. How much must be endorsed on each note in order to pay them off at the same time? What must be the endorsement on each if $a=1900$, $b=1800$, $m=6$, $n=7$, and $c=25$.

[This problem is the same as No. 86, Miscellaneous. See the solutions in that department.]

109. Proposed by B. F. FINKEL, A.M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

$\dfrac{1}{\sqrt{(x+1)}}+\dfrac{1}{\sqrt{(x-1)}}=\dfrac{1}{\sqrt{(x^2-1)}}$; find value of x satisfying the equation.

Solution by the PROPOSER.

This problem was proposed for the purpose of explaining the singular fact (singular to those who do not possess more than a mechanical knowledge of algebra) that the value of the unknown does not satisfy the original equation.

By transposing and factoring, the original equation may be written

$$\frac{1}{\sqrt{(x^2-1)}}[\sqrt{(x-1)}+\sqrt{(x+1)}-1]=0,$$

which is equivalent to the system of equations,

$$\left\{\begin{array}{ll}\dfrac{1}{\sqrt{(x^2-1)}}=0\ \ldots\ldots\ldots\ldots\ldots\ldots & A.\\ \sqrt{(x-1)}+\sqrt{(x+1)}-1=0\ \ldots\ldots\ldots & B.\end{array}\right\}$$

The solution of A gives $x=\infty$, which value satisfies the original equation.

From B we have $\sqrt{(x+1)}=1-\sqrt{(x-1)}$. Squaring both sides, transposing and combining, we have $1=-2\sqrt{(x-1)}$.

From this last, by squaring, we obtain $1=4(x-1)$, from which we find,

$x=\frac{5}{4}$. This value of x does not satisfy B, neither does it satisfy the original equation.

By studying the two operations of squaring, in the solution of B, it will be seen that extraneous equations were introduced. The squaring the first time was equivalent to multiplying B by $\sqrt{(x-1)}+\sqrt{(x+1)}+1$, that is, the equation resulting from squaring the first time is equivalent to

$$[\sqrt{(x-1)}+\sqrt{(x+1)}+1][\sqrt{(x-1)}+\sqrt{(x+1)}-1]=0\ldots.(1),$$

and the second operation of squaring which gives the equation $1=4(x-1)$ is equivalent to multiplying (1) by $[\sqrt{(x-1)}-\sqrt{(x+1)}+1][\sqrt{(x-1)}-\sqrt{(x+1)}-1]$, that is, the equation $1=4(x-1)$ is equivalent to the equation

$$[\sqrt{(x-1)}+\sqrt{(x+1)}+1][\sqrt{(x-1)}+\sqrt{(x+1)}-1]$$
$$[\sqrt{(x-1)}-\sqrt{(x+1)}+1][\sqrt{(x-1)}-\sqrt{(x+1)}-1]=0.$$

This equation, therefore, is equivalent to the system of equations

$$\left\{\begin{array}{l}\sqrt{(x-1)}+\sqrt{(x+1)}+1=0=P\\ \sqrt{(x-1)}+\sqrt{(x+1)}-1=0=Q\\ \sqrt{(x-1)}-\sqrt{(x+1)}+1=0=R\\ \sqrt{(x-1)}-\sqrt{(x+1)}-1=0=S\end{array}\right\}$$

Of these, the only one that is satisfied by the value $x=\frac{5}{4}$ is R. The solution, therefore, of any one of these equations gives the same value of x, viz., $x=\frac{5}{4}$, and this value of x satisfies R only.

This whole subject of *derivation of equations* has been neglected by most writers on algebra in America until quite recently. The recent texts of Fisher and Schwatt, Beman and Smith, and several others have given considerable attention to the subject.

Professor Chrystal has given a very good treatment of the subject in his Algebra, Vol. I, § XIV. On page 285, he says, "There are few parts of algebra more important than the logic of the derivation of equations, and few, unhappily, that are treated in a more slovenly fashion in elementary teaching."

This problem was also solved in a very excellent manner by *H. C. WHITAKER*, *G. B. M. ZERR*, *COOPER D. SCHMITT*. *W. H. CARTER*, *W. W. LANDIS*, *CHARLES C. CROSS*, *J. M. BOORMAN*, *J. D. CRAIG*, *A. F. KOVARIK*, *ELMER SCHUYLER*, and *J. SCHEFFER*.

110. **Proposed by J. C. CORBIN, Pine Bluff, Ark.**

Put down any number of pounds, shillings and pence under £11, taking care that the number of pence is less than the number of pounds. Reverse this sum, putting pounds in the place of pence, and subtract from the original. Again reverse this remainder and add. The result in all cases will be £12 18s 11d, neither more nor less, whatever the amount with which we start. Verify and explain.